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## **Pre-stressed cable truss with stiffening girder and design clearance: development and analysis**

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### **Abstract**

Cable truss systems possess a lot of advantages in comparison to ordinary concrete or steel structures. They are lightweight and able to span large free areas without intermediate supports. At the same time, they are very deformable due to, so-called, kinematic displacements. Horizontal thrust, brought about by cables, should be supported by bearer constructions of the building. It results in increase of material consumption and project expenditures in the whole. In order to equilibrate the thrust and to diminish the deflections of the structure stiffening girders with, so-called, design clearances are proposed to be used. The design clearance allows fine-tuning the cable truss in order to bear external loads in the most efficient way according to material properties of its principal elements. Computational technique for development and analysis of pre-stressed cable truss with stiffening girder and design clearance is given. The results, obtained by the proposed formulas are in a good agreement with data, provided by the specialized software of non-linear structural analysis Easy. The technique may be used either to perform preliminary design or to prepare initial data for software packages of non-linear analysis in order to simulate structural behavior under complex loading conditions. The present work contributes to further development of hybrid roofs.

**Keywords:** cable truss, stiffening girder, design clearance, hybrid roof, pre-stress

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## 1. Introduction

Cable truss systems consist of flexible chords, linked together by struts. They are lightweight and able to span large free areas without intermediate supports. Cable trusses exhibit high load bearing capacity and can be mounted in a short period of time. On the other hand, they are very deformable, especially in the event of non-uniform external loading due to, so-called, kinematic displacements (Wagner, 2016). It significantly reduces the efficiency of high-strength cables. In addition, horizontal thrust, brought about by a cable truss, should be supported by bearer constructions of the building, resulting in substantial increase of project expenditures.

In order to equilibrate the thrust, stiffening girders are used. When connected with the struts, they also reduce deflections of the truss (figure 1). On the other hand, the girder, made of ordinary structural steel, takes the major portion of the load, because its stiffness is substantially higher, in comparison to high-strength cables, which remain underused.

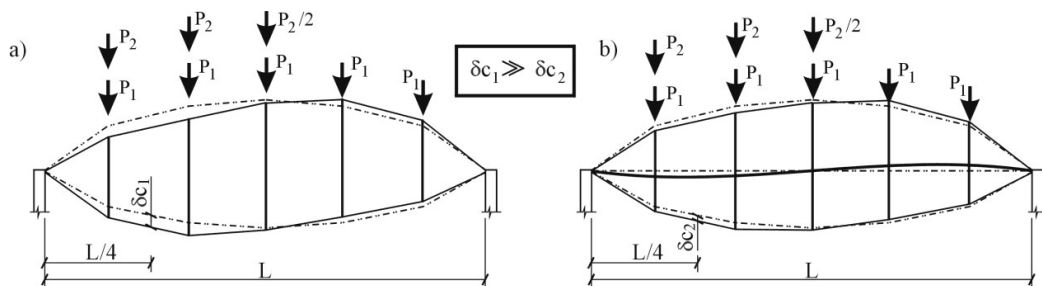


Figure 1: Ordinary cable truss (a) and cable truss with the stiffening girder (b).  
(Chesnokov, Mikhailov and Sabitov, 2010)

In order to ensure maximum efficiency of high-strength cables, so-called, design clearances are proposed to be used (Chesnokov and Mikhailov, 2010). The clearance is a gap, which allows the struts of the truss to move freely a certain distance in the vertical direction. While deformations of the cable roof are smaller, than the designed value of the clearance, the stiffening girder does not support the struts. It is loaded only by longitudinal thrust until excessive deformations occur. The girder begins taking transverse load preventing the truss from kinematic displacements. Thus, the design clearance allows fine-tuning the truss in order to bear external loads in the most efficient way, according to material properties of its principal elements.

Computational technique for obtaining stiffness properties of cables and the girder, required pre-stress values and the size of the design clearance is given in the present paper. It is derived from the analytical model of a single cable. Numerical example is considered in order to exhibit and analyze structural behavior of the truss.

The technique may be used either to perform preliminary design of the truss, or to provide initial data for non-linear structural analysis software in order to simulate the influence of all possible load-cases.

The present work contributes to further development of hybrid roof structures, made of flexible and rigid members. It extends the scope of high-strength structural steel for roofs of buildings.

## 2. Description of the structure

The truss (Chesnokov and Mikhailov, 2010) consists of bearer 1 and restraining 2 chords, made of high-strength cables, spreaders 3, stiffening girder 4 and supports 5 (figure 2). The bearer chord is equipped with tensioning devices, e.g. hydraulic jacks, grip hoists or turnbuckles, which allow to implement preliminary stresses in the truss (Seidel, 2009). The

spreaders 3 and the girder 4 are linked together by means of the design clearance. Retainers (figure 3), rigidly fastened to the spreaders 3, are inserted into the gaps. The length of the gap depends on the required size of the design clearance.

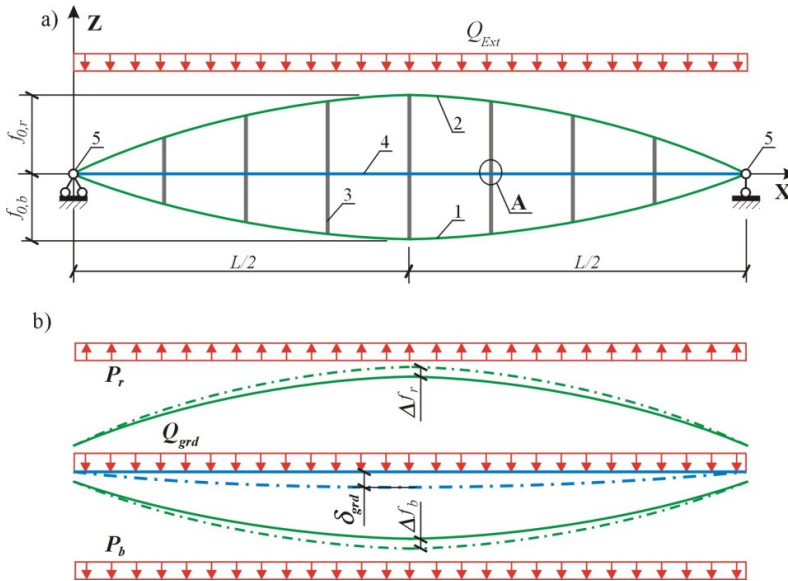


Figure 2: Cable truss with stiffening girder.

a – general view; b – loads on the cables and the girder.

1 – bearer chord; 2 – restraining chord; 3 – spreader; 4 – the stiffening girder; 5 – support; solid line shows initial configuration, while dash-dotted line is used for deformed configuration of the cables and the girder; joint A is illustrated in figure 3

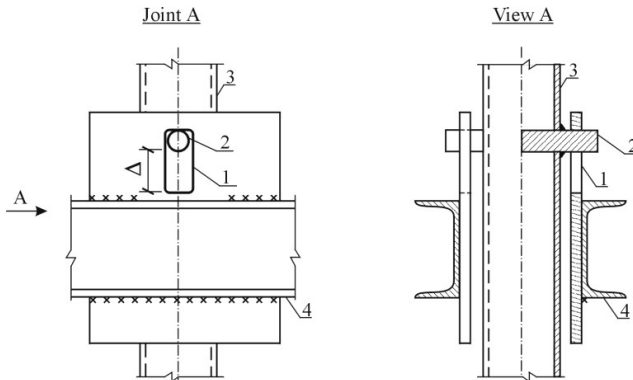


Figure 3: Joint A in figure 2.

1 – the design clearance; 2 – retainer; 3 – spreader of the cable truss; 4 – stiffening girder

### 3. Analysis of the structure

#### 3.1. General considerations

The behavior of the structure is analyzed in two stages. On the first stage the girder is not connected to the spreaders, and the cable truss is deformed by means of the bearer chord tensioning. Comparatively negligible bending moments arise in the girder, subjected by its own weight in the transverse direction. When the required level of pre-stressing of the truss is achieved the girder is united with the spreaders as shown in figure 3.

On the second stage of structural behavior of the truss operational load  $Q_{Ld}$  is applied on the top chord. It results in deflection of the cable truss until the design clearance is exhausted. Then, the girder takes transverse load  $Q_{grd}$ . It confines the total deflection of the truss by a limiting value  $\omega_{lim}$ , which should be provided from the serviceability and operability considerations.

General conditions (1) - (3) for the cable truss may be written:

$$\Delta f_r = -\Delta f_b \quad (1)$$

where  $\Delta f_v$  is the deflection of the corresponding chord of the cable truss,  $\Delta f_v$ -value is measured relative to the initial configuration of the cable (figure 2); hereinafter index “v” is either “r” or “b”, where the index “b” refers to the bearer chord, and the index “r” – to the restraining chord;

$$Q_{Ext} + P_r = P_b + Q_{grd} \quad (2)$$

$$0.01 \leq \Theta_v \leq 1.0 \quad (3)$$

where  $P_v$  is the load acting on the chord;  $Q_{Ext}$  is an external load, acting on the truss;  $\Theta_v = \frac{N_v}{A_v \cdot R_{cab}}$  is a chord ratio;  $R_{cab}$  is the load-carrying capacity of steel cables, forming the chords;  $A_v$  is the cross section area of the chord;  $N_v$  is the force in the chord v:

$$N_v = EA_v \cdot \varepsilon_v \quad (4)$$

where  $EA_v$  is the stiffness of the chord;  $\varepsilon_v$  is the relative deformation of the chord, calculated according to (Chesnokov and Mikhaylov, 2017):

$$\varepsilon = \frac{\Psi_4 \cdot f^4 + \Psi_2 \cdot f^2 + L}{L_{c0}} - 1 \quad (5)$$

where  $f$  is the sag of the chord;  $L$  is the span of the truss;  $L_{c0}$  is the initial length of the chord;  $\Psi_2$  and  $\Psi_4$  are the coefficients, calculated according to (Chesnokov and Mikhaylov, 2017) for uniformly distributed load as follows:  $\Psi_2 \approx 2.67/L$  and  $\Psi_4 \approx -6.4/L^3$ .

Initial length of the restraining chord coincides with its geometric (index “g”) length  $L_{c0,r} = L_{g,r}$ . Pre-stress of the cable truss is implemented by means of tensioning of the bearer chord  $\Delta L_p$ . So, initial length of the bearer chord is the following:  $L_{c0,b} = L_{g,b} - \Delta L_p$ . Geometric lengths of the chords ( $L_{g,b}$  and  $L_{g,r}$ ) are calculated according to (Chesnokov and Mikhaylov, 2017) using their initial ordinates in the middle of the span ( $f_{0,b}$  and  $f_{0,r}$ , figure 2) and the coefficients  $\Psi_2$  and  $\Psi_4$ .

Equilibrium equation for a chord  $v$ , subjected to uniformly distributed load, is derived from (Mikhaylov and Chesnokov, 2017):

$$EA_v = P_v / \lambda_v \quad (6)$$

where

$$\lambda_v = 8 \cdot \frac{f_v}{L^2} \cdot \varepsilon_v \quad (7)$$

where  $\varepsilon_v$  is calculated by means of substituting the corresponding value  $f = f_v$  into (5).

### 3.2. The first stage of structural behavior: pre-stress of the cable truss

On the stage of the pre-stress of the cable truss the external load and the girder’s transverse load are omitted:  $Q_{Ext,pr} = 0$  and  $Q_{grd,pr} = 0$ . In accordance to (2):

$$P_{b,pr} = P_{r,pr} \quad (8)$$

Hereinafter index “*pr*” refers to the stage of the pre-stress of the cable truss. The restraining chord ratio reaches its maximum value (3):

$$\Theta_{r,pr} = 1.0 \quad (9)$$

According to (9), the deflection of the restraining chord, brought about by pre-stress of the truss, may be calculated as follows:

$$\Delta f_{r,pr} = \sqrt{\frac{\sqrt{\Psi_2^2 + 4 \cdot \Psi_4 \cdot [(R_{cab} / E_{cab} + 1) \cdot L_{c0,r} - L]} - \Psi_2}{2 \cdot \Psi_4}} - f_{0,r} \quad (10)$$

where  $E_{cab}$  is the modulus of elasticity of steel cables.

Equilibrium equation (6) may be written for the stage of the pre-stress of the truss as follows:

$$EA_r = P_{r,pr} / \lambda_{r,pr} \quad (11)$$

$$EA_b = P_{r,pr} / \lambda_{b,pr} \quad (12)$$

Parameters  $\lambda_{v,pr}$  are obtained from (7) by substituting the following ordinates for the corresponding chord:

$$f_r = f_{r,pr} = f_{0,r} + \Delta f_{r,pr} \quad (13)$$

$$f_b = f_{b,pr} = f_{0,b} - \Delta f_{r,pr} \quad (14)$$

### 3.3. The second stage of structural behavior: external load

The external load  $Q_{Ext,Ld}$  is equal to, so-called, operational load or payload  $Q_{Ld}$ . Hereinafter index “ $Ld$ ” refers to uniformly distributed load on the cable truss. The girder’s transverse load is expressed from (2):

$$Q_{grd} = Q_{Ld} + P_{r,Ld} - P_{b,Ld} \quad (15)$$

The bearer chord ratio reaches the maximum value (3) when the external load acts on the truss:

$$\Theta_{b,Ld} = 1 \quad (16)$$

The pre-tension  $\Delta L_p$  of the bearer chord is derived from the condition (16):

$$\Delta L_p = L_{g,b} - L_{Ld,b} \cdot \frac{E_{cab}}{E_{cab} + R_{cab}} \quad (17)$$

where  $L_{Ld,b}$  is the length of the bearer chord of the loaded truss, calculated according to (Chesnokov and Mikhaylov, 2017), using the ordinate in the middle of the span:

$$f_b = f_{b,Ld} = f_{0,b} - \Delta f_{r,Ld} \quad (18)$$

where the deflection  $\Delta f_{r,Ld}$  of the truss under load is confined by the given limiting value  $\omega_{lim}$ :

$$\Delta f_{r,Ld} = \Delta f_{r,pr} - \omega_{lim} \quad (19)$$

In order to fulfill the left-hand side condition of (3) the limiting value for the deflection of the truss  $\omega_{lim}$  should be confined as follows:

$$\omega_{lim} \leq \Delta f_{r,pr} + \sqrt{\frac{\Psi_2^2 + 4 \cdot \Psi_4 \cdot \left( \frac{E_{cab} + R_{cab}}{E_{cab} \cdot (1 + \varepsilon_{lim})} \cdot L_{pr,b} - L \right) - \Psi_2}{2 \cdot \Psi_4}} - f_{0,b} \quad (20)$$

and

$$\omega_{lim} \leq \Delta f_{r,pr} - \sqrt{\frac{\Psi_2^2 + 4 \cdot \Psi_4 \cdot \left( (1 + \varepsilon_{lim}) \cdot L_{c0,r} - L \right) - \Psi_2}{2 \cdot \Psi_4}} + f_{0,r} \quad (21)$$

where  $\varepsilon_{lim} = 0.01 \cdot R_{cab} / E_{cab}$  is the minimum allowable relative tension of a cable chord;  $L_{pr,b}$  is the length of the bearer chord of the pre-stressed truss, calculated according to (Chesnokov and Mikhaylov, 2017), using the ordinate in the middle of the span (14).

Equilibrium equation (6) is written for bearer and restraining chords of the loaded truss as follows:

$$EA_r = P_{r,Ld} / \lambda_{r,Ld} \quad (22)$$

$$EA_b = P_{b,Ld} / \lambda_{b,Ld} \quad (23)$$

The parameters  $\lambda_{v,Ld}$  are obtained from (7) by substituting the ordinate for the bearer chord (18) and the corresponding ordinate for the restraining chord:

$$f_r = f_{r,Ld} = f_{0,r} + \Delta f_{r,Ld} \quad (24)$$

By means of equating (11) and (22), and also (12) and (23) the expressions for loads, acting on the chords of the truss in the operational stage, are derived as follows:

$$P_{r,Ld} = \frac{\lambda_{r,Ld}}{\lambda_{r,pr}} \cdot P_{r,pr} \quad (25)$$

$$P_{b,Ld} = \frac{\lambda_{b,Ld}}{\lambda_{b,pr}} \cdot P_{r,pr} \quad (26)$$

The load  $Q_{grd}$ , obtained from (15), should be positive, otherwise the girder doesn't take transverse load from the cable truss ( $Q_{grd} = 0$ ), and the construction splits into two sub-structures (the truss and the girder), linked together by longitudinal thrust only. From the condition  $Q_{grd} > 0$  and expressions (15), (25), (26) the upper bound for the load  $P_{r,pr}$  is obtained:

$$P_{r,pr} < P_{r,lim} = \frac{Q_{Ld}}{\frac{\lambda_{b,Ld}}{\lambda_{b,pr}} - \frac{\lambda_{r,Ld}}{\lambda_{r,pr}}} \quad (27)$$

The deflection of the girder, influenced by transverse and longitudinal loads is calculated according to (Galambos and Surovek, 2008):

$$\delta_{grd} = \frac{\delta_Q}{1 - N_{grd} / N_{el}} \quad (28)$$

under the condition:

$$N_{grd} < N_{el} \quad (29)$$

where  $\delta_Q$  is the deflection, brought about by the transverse load only, obtained from the differential equation of a beam (Fertis, 2006);  $N_{el}$  is the Euler load and  $N_{grd}$  is the axial force in the girder, equal to the thrust of steel cables:

$$\delta_Q = \frac{5}{384} \cdot Q_{grd} \cdot \frac{L^4}{E_{grd} \cdot I_{grd}} \quad (30)$$

$$N_{el} = \pi^2 \cdot E_{grd} \cdot I_{grd} / L^2 \quad (31)$$

$$N_{grd} \approx N_{r,Ld} + N_{b,Ld} \quad (32)$$

where  $N_{v,Ld}$  is the force in the chord  $v$  of the truss, calculated from (4), using the corresponding ordinate  $f_{v,Ld}$  (18), (24);  $E_{grd}$  is the modulus of elasticity of girder's material;  $I_{grd}$  is the moment of inertia of the girder's cross section.

Required value of the design clearance in the middle of the span is calculated from (33) (Chesnokov, Mikhailov and Sabitov, 2010). The clearance  $\Delta$  should be positive.

$$\Delta = \omega_{lim} - \delta_{grd} \quad (33)$$

The strength properties of the girder should be verified by the following criterion:

$$\sigma_{grd} < R_{grd} \quad (34)$$

where  $R_{grd}$  is the strength of the material of the girder and  $\sigma_{grd}$  is the stress in the girder:

$$\sigma_{grd} = \frac{N_{grd}}{A_{grd}} + \frac{M_{grd}}{W_{grd}} \quad (35)$$

where  $A_{grd}$  and  $W_{grd}$  are the cross section area and the elastic section modulus of the girder;  $M_{grd}$  is bending moment in the girder:

$$M_{grd} = \frac{Q_{grd} \cdot L^2}{8} + N_{grd} \cdot \delta_{grd} \quad (36)$$

#### 4. Case study

Cable truss, which consists of two flexible chords, linked together by vertical spreaders and horizontal girder, is considered (figure 2). The span of the truss is  $L=12$  m. Initial ordinates of the chords are the following:  $f_{0,b}=1.5$  m and  $f_{0,r}=1.0$  m. The chords consist of steel cables. The strength of the cables is  $R_{cab}=700$  MPa and the modulus of elasticity is  $E_{cab}=1.3 \cdot 10^5$  MPa (PFEIFER, 2017). The spreaders and the girder are made of structural steel with the following properties:  $R_{grd}=210$  MPa and  $E_{grd}=2.1 \cdot 10^5$  MPa.

The deflection of the truss under load should be confined by the following values (20), (21):  $\omega_{lim} \leq \min(0.112, 0.141)$  m. It is adopted  $\omega_{lim} = L/120 = 0.1$  m, while the external load is taken equal to  $Q_{Ld} = 25.0$  kN/m.

The bearer chord pre-tension is  $\Delta L_p = 0.0932$  m (17), while cambering of the truss on the stage of pre-stress (10) is  $\Delta f_{r,pr} = 0.143$  m. The load between the chords of the truss on the stage of the pre-stress is taken  $P_{r,pr} = 1.1$  kN/m. It obeys the condition (27), where  $P_{r,lim} = 2.96$  kN/m.

Cable cross section areas, calculated from (11) and (12), are the following: the bearer chord  $A_b = 1.69 \text{ cm}^2$  and the restraining chord  $A_r = 0.25 \text{ cm}^2$ .

The forces in the chords are calculated for the stage of the pre-stress  $N_{b,pr} = 14.6 \text{ kN}$ ,  $N_{r,pr} = 17.3 \text{ kN}$ , and for the stage of the external load  $N_{b,Ld} = 118.1 \text{ kN}$ ,  $N_{r,Ld} = 5.0 \text{ kN}$ . Corresponding  $\Theta$ -ratios obey the condition (3): 0.12, 0.99, 0.99 and 0.29.

Transverse (15) and longitudinal (32) loads on the girder are the following:  $Q_{grd} = 15.7 \text{ kN/m}$  and  $N_{grd} = 123.1 \text{ kN}$ . The girder is made of two channel bars (figure 3). It has the following parameters:  $A_{grd} = 123.0 \text{ cm}^2$ ,  $I_{grd} = 30440 \text{ cm}^4$  and  $W_{grd} = 1493 \text{ cm}^3$ . The deflection of the girder (28) is  $\delta_{grd} = 0.068 \text{ m}$  and the design clearance (33) is  $\Delta = 0.032 \text{ m}$ .

Bending moment in the girder (36) is the following  $M_{grd} = 291 \text{ kN} \cdot \text{m}$ , while the stress in the girder is  $\sigma_{grd} = 205 \text{ MPa}$ . It obeys the condition (34).

The cable truss was analyzed by the module of non-linear structural simulation BEAM of the specialized software package Easy. Results, provided by Easy, are very close to the ones, calculated by the proposed formulas:  $\Delta f_{r,pr} = 0.144 \text{ m}$ ,  $N_{b,pr} = 14.3 \text{ kN}$ ,  $N_{r,pr} = 16.9 \text{ kN}$ ,  $N_{b,Ld} = 120.9 \text{ kN}$ ,  $N_{r,Ld} = 4.02 \text{ kN}$ ,  $\delta_{grd} = 0.066 \text{ m}$ ,  $N_{grd} = 121.0 \text{ kN}$ ,  $M_{grd} = 291 \text{ kN} \cdot \text{m}$  and the total deflection of the truss under load is  $\omega = 0.098 \text{ m}$ .

The effect of the stiffening girder is illustrated in figure 4, where  $\Delta f_{1/2}$  and  $\Delta f_{1/4}$  are the deflections of the structure in the middle and in the quarter of the span, respectively,  $K_{Ld}$  is the factor of load nonuniformity and  $\zeta_{1/2}$ ,  $\zeta_{1/4}$  are relative deflections  $\zeta^i = \Delta f^i / \Delta f^0$ , where  $\Delta f^0$  is the deflection under uniform load ( $K_{Ld} = 0$ ).

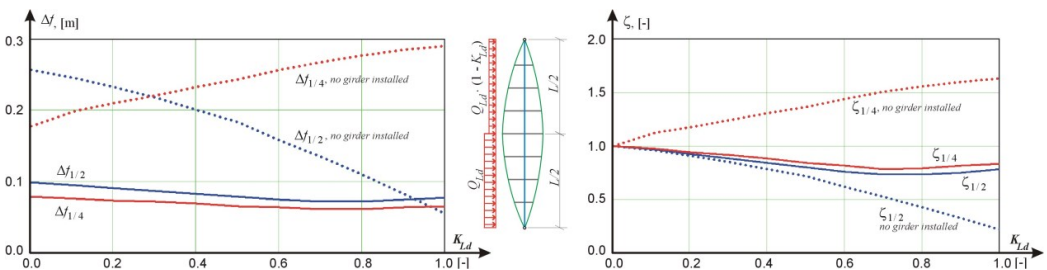


Figure 4: Deflections of the cable truss, obtained by the specialized software Easy (remark: both supports of the truss are fixed when the girder is not installed)

The figure 4 shows, that if the girder is installed, the deflections of the truss are substantially smaller, than that of an ordinary cable truss without the girder. On the other hand, non-linearity of curves, illustrated in figure 4, points to the necessity of asymmetrical load inclusion into the computational technique proposed.

## 5. Conclusion

Pre-stressed cable truss with stiffening girder and design clearance is considered. Its primary advantages in comparison to ordinary cable structures are smaller deformations under load, self-equilibration of the thrust and efficiency of material consumption.

Computational technique for structural analysis of the truss is given. It ensures to achieve preset deflection of the truss using material properties to the full extent. The technique includes simulation of the stage of the pre-stress and operational stage of the truss. It allows fine-tuning of the main parameters in order to increase the truss competitiveness and to highlight its advantages.

The technique contributes to structural optimization of the truss with stiffening girder. It may be used to perform preliminary design of the truss and to provide initial data for software packages of non-linear analysis in order to simulate complex behavior under different loading conditions.

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