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Numerical technique for estimation of cable roof structural parameters

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Abstract

Cable roof with reduced overall height is proposed for long-span buildings. It is more attractive from an economic point of view due to diminishing of unused internal space. Bearer cables of the roof are subdivided into primary and ordinary. Primary cables are arranged far from each other. They are directly connected to columns of the building. Ordinary cables are supported by primary ones. The distance between them is comparatively small. Specialized software systems for nonlinear analysis require the basic parameters of the construction to be determined before structural simulation. Numerical technique for estimation of pre-stress values and stiffness properties of elements of the construction is proposed in the present paper. The coordinate descent method is used to perform structural optimization. This approach allows to gain precise analytical results proved by the comparison with data, provided by the non-linear software package EASY. The present study contributes to the improvement and further development of cable and membrane roofs of long-span buildings, particularly in the field of industrial construction. It facilitates structural simulation by means of providing appropriate initial data for computer systems of non-linear static analysis.

Keywords: cable roof, reduced overall height, coordinate descent method, numerical technique

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1. Introduction

Cable roofs are widely used for large-span building and constructions. They allow to maximize the distance between internal supports, resulting in huge unobstructed space inside the building.

On the other hand, cable structures are very deformable, especially in the event of non-uniformly distributed external loads. Rigid roof elements, supported by cables, may get damages due to excessive deformations. Thus, flexible polymer membrane, made of polyester or glass fibers, covered with PVC, teflon or silicon (Houtman, 2003), is more attractive to be applied with cable structures. In addition, polymer membrane is lightweight and translucent. It allows to diminish installation and operational costs of the building.

Cable and flexible membrane structures are primarily confined by the field of civil engineering. They are successfully used in stadiums (Grunwald and Seethaler, 2011; Goppert, 2013), railway stations, fair sites, etc. Industrial buildings also need to be covered by such unsurpassed structures. They, however, require to reduce unused internal spaces and to diminish operational expenditures. So, approximately flat roof is much more appropriate for industrial construction in contrast to public buildings, which primarily need impressive architectural appearance.

Cable roof structure, considered in the present paper, consists of bearer and backstay cables, spreaders and ties (Chesnokov and Mikhaylov, 2016; Mikhaylov and Chesnokov, 2017) (figures 1 and 2).

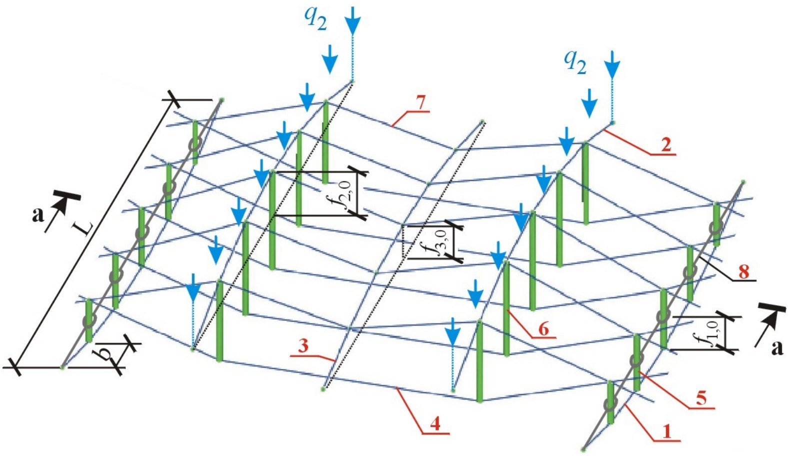

Figure 1: Axonometric view of a section of the roof
(Mikhaylov and Chesnokov, 2017).

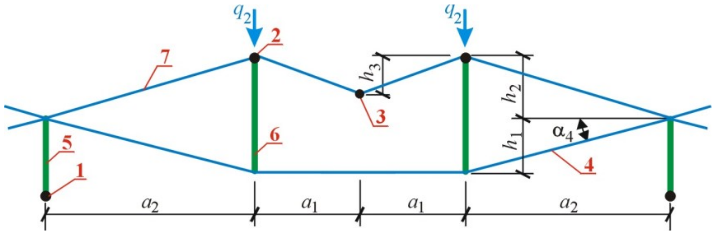

Figure 2: View of the roof along the line a-a in figure 1 (Mikhaylov and Chesnokov, 2017).

Bearer cables are arranged in mutually perpendicular directions. Longitudinal bearer cables 1 are attached fixedly to columns. They are situated far from each other, allowing large free areas to be ensured inside the building. Transverse bearer cables 4 are connected to struts 5, which are, in turn, supported by the cables 1 and fixed by means of ties 8 in the longitudinal direction. Backstay cables 2, supported by struts 6, are convex upwards. Together with ties 3 and 7, they form the top chord of the roof.

The overall height of the roof is smaller in comparison to ordinary cable-membrane structures of the same free span. Membrane curvature, however, is ensured to be in the appropriate range. It allows the membrane to sustain external loads without wrinkles and slackened areas (Forster, 2003).

2. Analysis of the structure

2.1. General considerations

The following parameters of the cable roof structure are taken into account:

- the modulus of elasticity and the strength of the cables and the struts E_{cab}, R_{cab} and E_{str}, R_{str} respectively;
- cross section areas of the cables A_j ($j = 1, \ 2, \ 3, \ 4$), diameters of the struts D_5, D_6 and their thickness-to-diameter ratios $k_{t,5}$, $k_{t,6}$;
- cable tensioning $\Delta L_{p,1}$, $\Delta L_{p,2}$ and $\Delta L_{p,3}$;
- the span of the primary bearer cables L, the distances between the cables a_1, a_2 and b, initial sags of the cables $f_{i,0}$ and height dimensions h_i ($i = 1, \ 2, \ 3$).

Cable tensioning $\Delta L_{p,i}$ is implemented by means of an appropriate pre-stressing equipment (Seidel, 2009). It is equal to the difference between, so called, geometrical length of the cable $Lc_{i,g}$ and the length of the cable $Lc_{i,0}$ in the unstressed state: $\Delta L_{p,i} = Lc_{i,g} - Lc_{i,0}$. The geometrical length of the cable $Lc_{i,g}$ is determined by its shape in the initial state, excluding any deflection of the structure: $\Delta f = 0$.

Numbering of the structural elements is in the figures 1 and 2. Although the pre-stress may be performed by means of the bottom cables 1 only (Mikhaylov and Chesnokov, 2017), tensioning $\Delta L_{p,3}$ of the tie 3 results in more lightweight structure, due to the possibility of force adjustment, which prevents the top chord from slackening. On the other hand, embedding tensioning equipment into the cables 2 is not justified, so the value $\Delta L_{p,2}$ is assumed equal to zero.

Cable cross section areas $\vec{X} = (A_1,\ A_2,\ A_3,\ A_4)^T$ and cable tensioning $\vec{\Delta L}_p = (\Delta L_{p,1},\ \Delta L_{p,3})^T$ are taken as independent parameters, which are to be determined by means of a structural optimization technique. Struts diameters D_5 and D_6 are also not preliminary known, but they can be derived from the buckling prevention condition of compressed structural members.

2.2. Flexible steel cable elements

Operability condition of the steel cables, constituting the roof structure, is taken into account as follows:

$$\Theta_{\lim,1} \le \Theta_j \le \Theta_{\lim,2} \tag{1}$$

where Θ_j is the ratio (2), while $\Theta_{\lim,1}$ and $\Theta_{\lim,2}$ are the limits of allowable range for the ratio:

$$\Theta_j = \frac{N_j}{A_j \cdot R_{cab}} \tag{2}$$

where N_j is the force in the cable *j*.

The left-hand side of the condition (1) ensures, that the cable *j* is in tension and does not slack under load, while the right-hand side prevents the cable from overstress.

The forces N_j are calculated according to (Mikhaylov and Chesnokov, 2017), considering $A_7 = A_8 = A_3$ and $q_1 = q_3 = 0$. They are obtained for the stage of pre-stress of the structure $N_{j,pr}$ and for, so-called, operational stage $N_{j,Ld}$, separately.

It is assumed, that the external load q_2 acts along the cable 2 from top to bottom, as shown in the figures 1 and 2. For the operational stage the load is brought about by the structural own weight, snow and suspended equipment: $q_2 = q$. On the stage of the pre-stress the external load is omitted due to its insignificance: $q_2 = 0$.

The force N_4 in the ordinary bearer cable 4 is obtained from the expression, given in (Mikhaylov and Chesnokov, 2017), using uniformly distributed loads p_i, acting on the cables ($i = 1,\ \ 2,\ \ 3$). The forces in the primary bearer cable N_1, the backstay cable N_2 and the longitudinal tie N_3 are obtained from the Hook's law:

$$N_i = E_{cab} \cdot A_i \cdot \varepsilon_i \tag{3}$$

where ε_i is the relative deformation of the cable, derived from (Chesnokov and Mikhaylov, 2017):

$$\varepsilon_i = \varepsilon_i(\Delta f_i, \Delta L_{p,i}) = \frac{\Psi_4 \cdot f_i^4 + \Psi_2 \cdot f_i^2 + L}{Lc_{i,0}} - 1 \tag{4}$$

where Ψ_2 and Ψ_4 are the coefficients, calculated for the middle of the span of the cable: $\Psi_2 \approx 2.67 / L$ and $\Psi_4 \approx -6.4 / L^3$; $f_i = f_{i,0} + \Delta f_i$ is the cable sag; Δf_i is the deflection of the cable in the middle of the span, obtained from (Mikhaylov and Chesnokov, 2017); $Lc_{i,0}$ is the length of the cable in the unstressed state: $Lc_{i,0} = Lc_{i,g} - \Delta L_{p,i}$.

It is proposed to confine deformations of the roof under load by the following condition:

$$\Delta f_{2,pr} - \Delta f_{2,Ld} \le \omega_{\lim} \tag{5}$$

where $\omega_{\lim}$ is the limit value of the structural deflection; $\Delta f_{2,pr}$ and $\Delta f_{2,Ld}$ are the deflections of the cable 2 brought about by pre-stressing of the roof and by the external load, respectively.

2.3. Compressed structural members

Diameters of the struts D_5 and D_6 are obtained from the conditions of compressive buckling (6) and flexibility limitation (7):

$$N_{\nu} \leq N_E \tag{6}$$

$$\lambda_{\nu} \leq \lambda_{lim} \tag{7}$$

where the index ν is either 5 or 6, depending on the strut considered, N_E is the Euler load (Galambos and Surovek, 2008), λ_ν is the slenderness of the strut, $\lambda_{\lim} = 120$ is the limit slenderness, N_ν is the force in the corresponding strut:

$$N_5 = p_1 \cdot b \tag{8}$$

$$N_6 = N_4 \cdot \sin(\alpha_4) \tag{9}$$

Conditions (6) and (7) allow to derive the strut diameter:

$$D_\nu \geq \sqrt{L_\nu} \cdot \sqrt[4]{\frac{8 \cdot N_\nu}{\pi^3 \cdot E_{str} \cdot k_{t,\nu} \cdot \left(1 - 3 \cdot k_{t,\nu} + 4 \cdot k_{t,\nu}{}^2 - 2 \cdot k_{t,\nu}{}^3\right)}} \tag{10}$$

$$D_\nu \geq \frac{2 \cdot L_\nu}{\lambda_{\lim} \cdot \sqrt{k_{t,\nu}{}^2 - k_{t,\nu} + 0.5}} \tag{11}$$

where L_ν is the length of the strut.

3. Structural optimization of the cable roof

Parameters of the cable roof form a multidimensional space, which consists of two sub-spaces. The first one contains permissible parameter combinations Ξ_p, while the second, or invalid sub-space Ξ_i describes the roof structure, which is either not operable or doesn't comply with the conditions imposed.

Permissible parameter groups Ξ_p are not equivalent. Additional requirements, such as material consumption, expenditures, labor input etc., distinguish them from each other, resulting in the best combination, which should be determined. So, the problem of structural optimization arises.

Complex behavior of the cable roof, reflected in slackening of compressed members and instability of the structure in the whole, substantially complicates gradient optimization techniques and diminishes their effectiveness. Among derivative-free, or zero-order, approaches the coordinate descent method allows to gain reliable and numerically stable results for the problem considered. It is accepted as a basis for the technique, elaborated in the present work for estimation of cable roof structural parameters (figure 3).

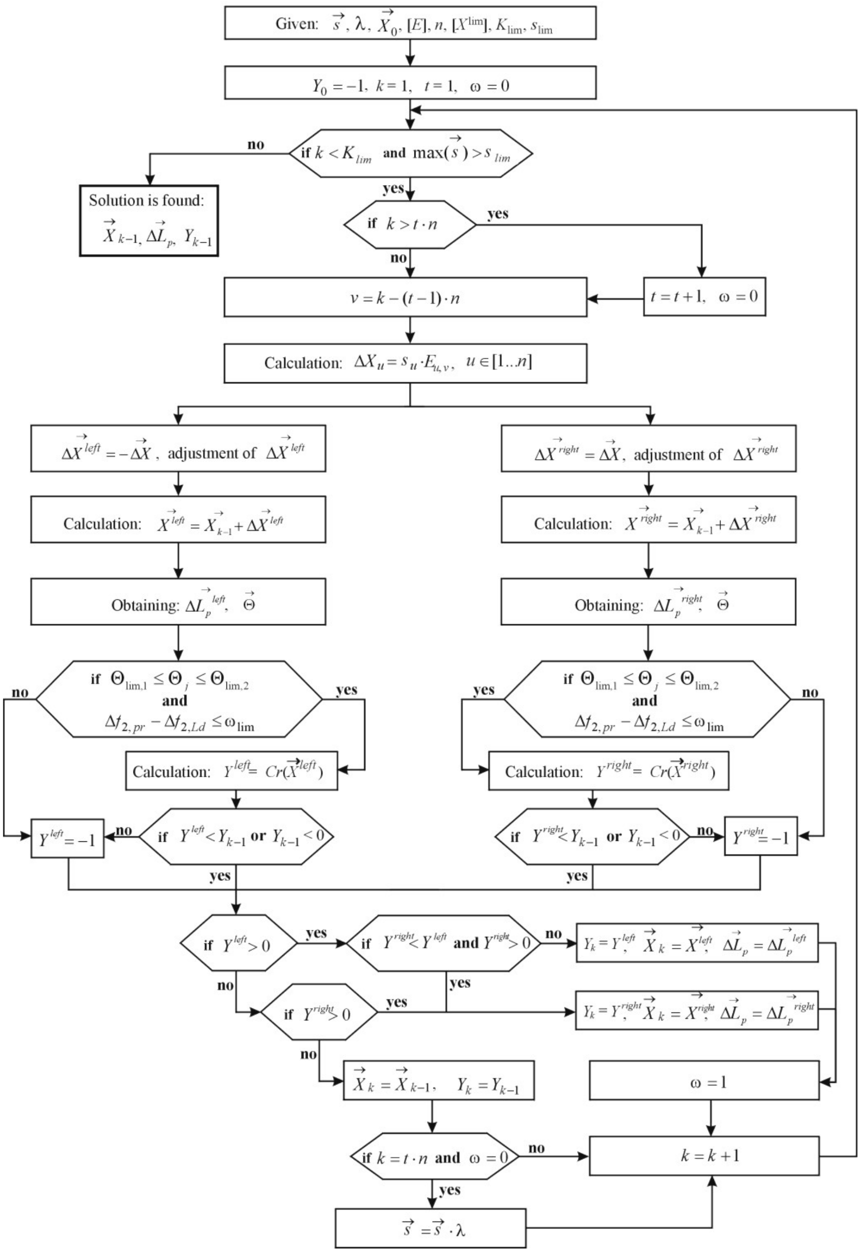

Figure 3: The technique for structural optimization of the cable roof

Parameters of the technique are the following: k is the number of current iteration and $K_{\lim}$ is the maximum number of iterations specified; $\vec{X}_0$ is the vector of initially given values of parameters to be optimized, while $\vec{X}_k$ contains current parameter values; $n = 4$ is the number of components of the vector $\vec{X}$; $[X^{\lim}]$ is the matrix, which consists of n rows and 2 columns, it contains allowable ranges for parameters to be optimized; $[E]$ is the identity matrix of n rows and columns; $\vec{s}$ is a vector of n components, which contains possible variations of the parameters; $s_{\lim}$ is a limiting value for a component of $\vec{s}$; λ is a reduction factor for $\vec{s}$.

The criterion function is taken as follows:

$$Cr(\vec{X}) = \Sigma M_{cab} + \Sigma M_{str} \cdot \rho_{str} \tag{12}$$

where ΣM_{cab} and ΣM_{str} are total masses of the cables and the struts belonging to the structure considered, ρ_{str} is the ratio of the average price of struts to the average price of cables.

According to the optimization technique only one component of the vector $\vec{X}$ is modified on each iteration step by means of decrementing $\overrightarrow{X^{left}}$ and incrementing $\overrightarrow{X^{right}}$ the current value. Variations of the vector $\vec{X}$ are adjusted in order that the resultant vector components would be in the appropriate range, specified by the matrix $[X^{\lim}]$.

Cable tensioning $\vec{\Delta L}_p = (\Delta L_{p,1},\ \ \Delta L_{p,3})^T$, corresponding to the current vector $\vec{X}$, is calculated in the step-by-step way (figure 4).

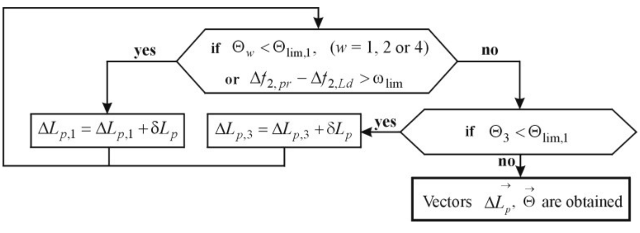

Figure 4: Obtaining cable tensioning $\vec{\Delta L}_p$

The initial components of the vector $\vec{\Delta L}_p$ are assumed equal to the increment $\Delta L_{p,1} = \Delta L_{p,3} = \delta L_p$. If the condition (5) or the left-hand side of the condition (1) are not satisfied, the increment δL_p is added to the corresponding component. However, if the right-hand side of the condition (1) becomes not valid for any cable j, the step-by-step process is terminated, and current $\vec{X}$-vector is considered unacceptable.

The criterion function value (12) is saved into variable Y (figure 3). Negative Y-value means either that current parameters are not permissible, or that the criterion function value is worse, than the previously obtained result.

4. Case study

4.1. Initial data

The initial data taken into account are the following: the moduli of elasticity $E_{cab} = 130$ and $E_{str} = 206$ GPa; the strength properties $R_{cab} = 700$ and $R_{str} = 240$ MPa; dimensions of the roof $L = 12.0$ m, $a_1 = 3.0$ m, $a_2 = 3.0$ m, $b = 2$ m, $h_1 = 1.0$ m, $h_2 = 1.5$ m, $h_3 = 0.8$ m; initial sags of the cables $f_{1,0} = 1.0$ m, $f_{2,0} = 1.0$ m and $f_{3,0} = 0.7$ m; thickness-to-diameter ratios of the struts $k_{t,5} = k_{t,6} = 1/20$; the external load $q = 10.8$ kN/m; the limiting value for the structural deflection $\omega_{\lim} = L/150 = 0.08$ m; the limiting values for the Θ_j-range (1): $\Theta_{\lim,1} = 0.1$ and $\Theta_{\lim,2} = 1.0$.

The matrix of admissible ranges for parameters to be optimized is the following:

$$X^{\lim} = \begin{bmatrix} A_{\lim,1} & A_{\lim,2} \\ A_{\lim,1} & A_{\lim,2} \\ A_{\lim,1} & A_{\lim,2}/5 \\ A_{\lim,1} & A_{\lim,2} \end{bmatrix} \quad (13)$$

where $A_{\lim,1} = 22$ mm^2 and $A_{\lim,2} = 1560$ mm^2 are the limiting cross section areas of the cables, corresponding to the interval, which ranges from one cable with the diameter 6.1 mm and up to two cables with the diameter 36.6 mm (PFEIFER, 2017).

Initial vector $\vec{s}$ of variations of parameters is obtained under the following expression: $s_u = (X^{\lim}{}_{u,2} - X^{\lim}{}_{u,1})/20$, where $u = 1...n$, $n = 4$. The limiting value for $\vec{s}$ is adopted the following: $s_{\lim} = 0.005$, while the maximum number of iterations is specified $K_{\lim} = 500$. The vector $\vec{X}_0$ of initially given values for parameters is obtained under the following expression:

$X0_u = (X^{\lim}{}_{u,1} + X^{\lim}{}_{u,2})/2$. The increment for the cable tensioning is adopted the following: $\delta L_p = 0.01$. The price ratio is taken the following: $\rho_{str} = 1/3$.

4.2. Comparison with results, obtained by the licensed software for non-linear structural analysis

The following $\vec{X}$-vector (1) is obtained by means of the iteration technique (figure 3):

$$\vec{X} = \begin{pmatrix} 4.42 & 2.14 & 0.22 & 1.54 \end{pmatrix}^T \text{ cm}^2 \tag{14}$$

The corresponding $\Delta\vec{L}_p$-vector is the following:

$$\Delta\vec{L}_p = (0.1248 \quad 0.0347)^T \text{ m} \tag{15}$$

Diameters of the struts, obtained from (10) and (11), are the following: $D_5 = 31$ mm and $D_6 = 62$ mm.

The criterion function value (12) is 254.7. The masses of cables and struts, belonging to the structure considered, are the following: $\Sigma M_{cab} = 213.6$ and $\Sigma M_{str} = 123.3$ kg.

In order to verify the proposed technique, structural analysis of the cable roof was performed by means of the specialized software package EASY. The comparison of results is accomplished by the following expression:

$$\xi = \frac{\left|\Lambda^e - \Lambda^p\right|}{0.5 \cdot \left|\Lambda^e + \Lambda^p\right|} \cdot 100 \tag{16}$$

where ξ is the relative discrepancy, %; Λ is the structural parameter to be compared; indexes "*e*" and "*p*" refer to the results, obtained by the EASY software and by the proposed formulations, respectively.

Comparison of forces in the cables $j = 1,\ 2,\ 3,\ 4$ and in the struts 5 and 6 are in the tables 1 and 2. The forces are given in kilonewtons. The deflections of the primary bearer cable 1 and the backstay cables 2, obtained in two ways, are very close to each other: $\Delta f^p{}_{1,pr} = -0.2024$ m, $\Delta f^p{}_{1,Ld} = -0.1493$ m, $\Delta f^p{}_{2,pr} = 0.1436$ m, $\Delta f^p{}_{2,Ld} = 0.0636$ m, and $\Delta f^e{}_{1,pr} = 0.2050$ m, $\Delta f^e{}_{1,Ld} = 0.1518$ m, $\Delta f^e{}_{2,pr} = 0.1495$ m, $\Delta f^e{}_{2,Ld} = 0.0690$ m. Corresponding discrepancies (16) are the following: $\xi_{1,pr} = 1.3$ %, $\xi_{1,Ld} = 1.7$ %, $\xi_{2,pr} = 4.0$ % and $\xi_{2,Ld} = 8.1$ %.

Remark: the deflections $\Delta f^{p}{}_{1,pr}$ and $\Delta f^{p}{}_{1,Ld}$ are multiplied by -1.0 before substitution into (16), because positive displacements of the cable 1, assumed in the present paper and in the EASY software are opposite to each other.

Table 1: Comparison of forces in the cables

Load-case	$N^{p}{}_{1}$	$N^{e}{}_{1}$	$\Theta^{p}{}_{1}$	$N^{p}{}_{2}$	$N^{e}{}_{2}$	$\Theta^{p}{}_{2}$	$N^{p}{}_{3}$	$N^{e}{}_{3}$	$\Theta^{p}{}_{3}$	$N^{p}{}_{4}$	$N^{e}{}_{4}$	$\Theta^{p}{}_{4}$
Pre-stress only	218.8	212.7	0.70	149.6	143.9	1.00	9.65	9.80	0.62	63.5	60.0	0.60
	$\xi_{1,pr}=2.8\%$			$\xi_{2,pr}=3.9\%$			$\xi_{3,pr}=1.6\%$			$\xi_{4,pr}=5.7\%$		
Pre-stress and vertical load q	309.1	301.8	1.00	64.1	59.5	0.43	1.54	1.68	0.10	92.6	86.0	0.86
	$\xi_{1,Ld}=2.4\%$			$\xi_{2,Ld}=7.4\%$			$\xi_{3,Ld}=8.7\%$			$\xi_{4,Ld}=7.4\%$		

Table 2: Comparison of forces in the struts

Load-case	$N^{p}{}_{5}$	$N^{e}{}_{5}$	$N_{E,5}$	λ_5	$N^{p}{}_{6}$	$N^{e}{}_{6}$	$N_{E,6}$	λ_6
Pre-stress only	19.37	19.26	31.7	95.9	20.1	19.9	81.1	120.0
	$\xi_{5,pr}=0.6\%$				$\xi_{6,pr}=1.0\%$			
Pre-stress and vertical load q	29.2	28.8			29.3	29.2		
	$\xi_{5,Ld}=1.4\%$				$\xi_{6,Ld}=0.3\%$			
Remark:	N_E and λ are designated in the expressions (6) and (7).							

5. Conclusion

Cable roof structure intended for large-span buildings is considered. In spite of comparatively reduced overall height, the roof ensures the membrane covering to be of a required curvature in order to avoid wrinkles and slackened areas.

Numerical technique, based on the coordinate descent method, is used to perform structural optimization. The technique is numerically stable and allows to gain reliable solutions for the problem, multidimensional parameter space of which includes, so-called, invalid sub-spaces Ξ_i. It allows to determine pre-stress values and stiffness properties of structural members, needed for automated computer simulation of the construction. In comparison to graphical approach, used in the previous work, the technique proposed in the present paper is much more effective, providing reliable results in a short period of time.

The future improvement of the optimization technique should be in the field of cost estimation refinement, including expenditures for manufacturing and installation of the cable roof in the whole and also particular joints and details.

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